

# Accuracy of Geometric Geoid Model of Singapore using RTK Heighting

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**Key words:** Geometric Geoid, RTK Heighting

## SUMMARY

A complete re-levelling of the precise levelling network of Singapore was carried out to derive the reduced levels of some 2000 plus benchmarks, which comprise new Precise Levelling Bench Marks (PLBMs), existing PLBMs, their witness marks and existing Integrated Survey Network (ISN) marks. A geometric geoid model for Singapore was also computed from the geoidal heights derived from the RTK ellipsoidal heights and the adjusted reduced levels of 406 marks which comprise PLBMs, witness marks, offset marks and ISN marks.

43 independent variables dependent on the easting and northing coordinates were initially used to compute the multiple regression surface to best fit 464 benchmarks with geoid separations (GS) which are the dependent variable in the formulation. 58 benchmarks were found to have outlying GS and subsequently found to have gross errors in their RTK heights. Recommendations had been made to avoid such spurious RTK heights. Stepwise multiple regression using the method of forward selection was used to compute the polynomial equation for the geometric geoid. Only five (5) independent variables and a constant term were ultimately adopted in the formulation.

The geometric geoid as determined using multiple regression forward stepwise method, RTK heighting and the precise leveling, is able to achieve accuracy to within  $\pm 0.030$  m,  $\pm 0.040$  m and  $\pm 0.050$  m for 82%, 95% and 99%, respectively, of the 464 benchmarks/offset marks/witness marks surveyed.

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## 1. INTRODUCTION

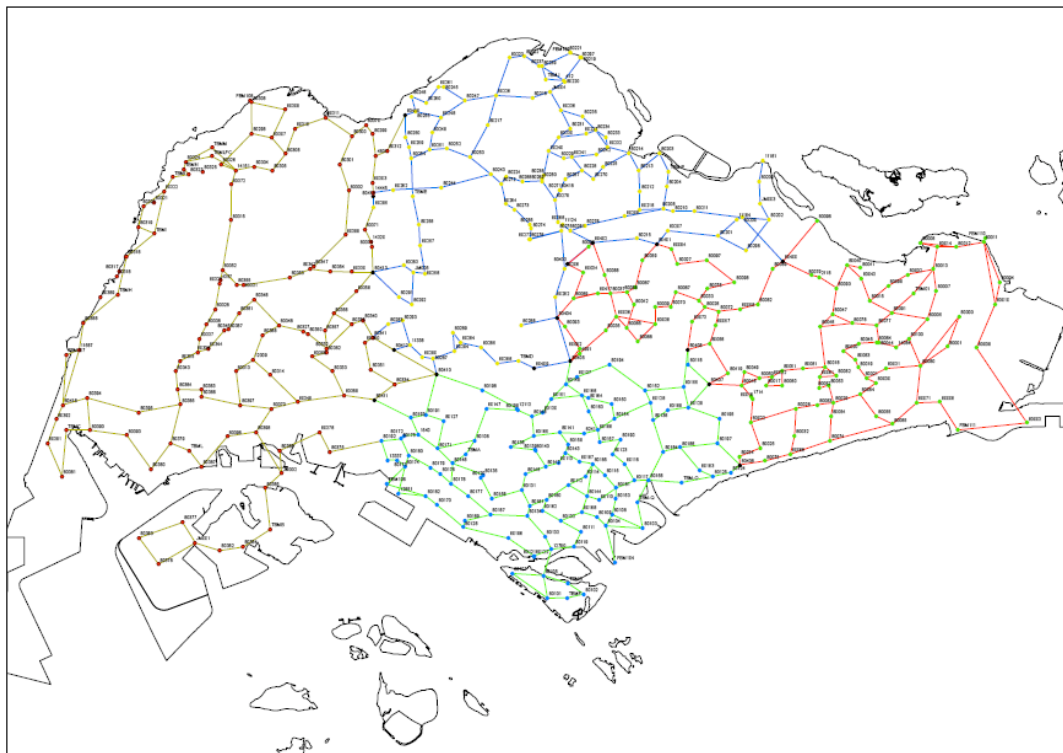
A Geometric Geoid model for Singapore was modelled using the geoidal heights derived from the ellipsoidal heights acquired using Global Positioning System (GPS) and the reduced levels of 406 Precise Level Bench Marks (PLBMs). These PLBMs cover the whole of Singapore mainland. 58 test benchmarks were used to verify the accuracy of the geoid model. This paper reports on the achieved accuracy of the reduced levels derived from the model.

## 2. RE-ESTABLISHING OF SINGAPORE PRECISE LEVELLING NETWORK

### 2.1 Singapore Precise Levelling Network

A complete re-levelling of the Singapore Precise Levelling Network was the first step in creating a Geoid Model of Singapore.

Figure 1 shows the overall layout of the levelling routes carried out in this project.

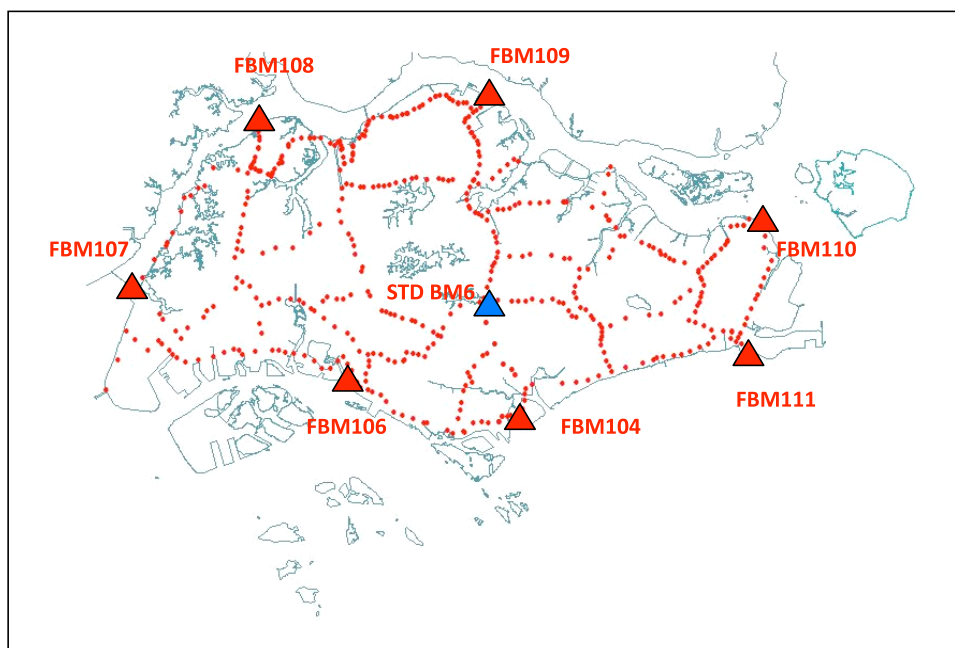


**Figure 1: Overall layout of levelling**

The levelling routes were organized into 4 batches for contracting purpose (SLA, 2009). Common benchmarks between the 4 batches were ensured so as to form a homogeneous levelling network covering the whole of Singapore mainland.

Two-way levelling was performed between two PLBMs to within the specification of  $\pm 2.5\sqrt{K}$  mm with K in km. GPS Real Time Kinematic (RTK) heighting of 20 readings with single intialisation per PLBM were also obtained.

Reduced levels of seven (7) Fundamental Benchmarks, i.e. FBM104, FBM106, FBM107, FBM108, FBM109, FBM110 and FBM111, are also determined in this project (Figure 2). STDBM6 was adopted as the reference benchmark and was confirmed with 116 other existing benchmarks with differences of within 3 mm.

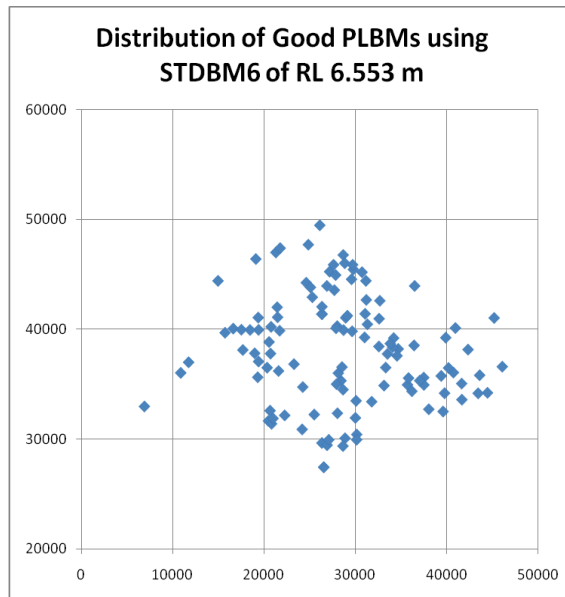


**Figure 2: Location of Fundamental Bench Marks and STDBM6**

## 2.2 Choice of Level Datum

Based on a minimum-constraint least squares adjustment computed using Move3D Levelling Least Squares program (Grontmij, 2009), it was found that adopting the reduced level of STDBM6 as 6.553 m (the pre-1997 value) gives better fit with 117 existing benchmarks to within 3 mm. On the other hand, only 40 existing benchmarks fit well to within 3 mm when using the post-1997 value of 6.541 m.

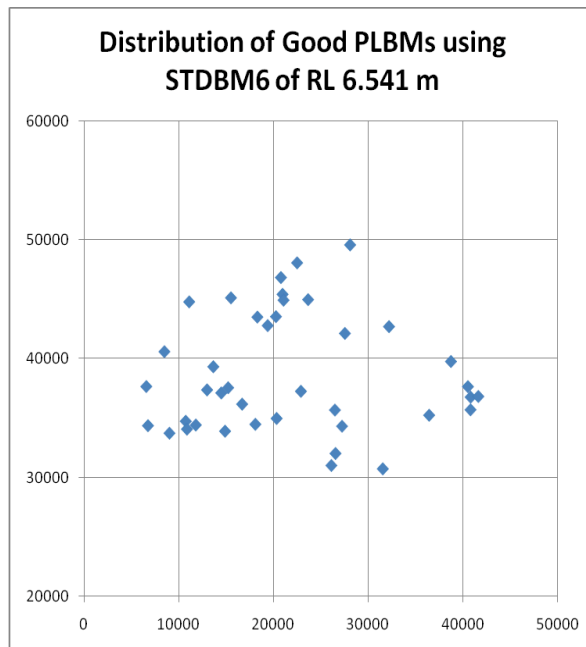
Figure 3 shows the distribution of the PLBMs using STDBM6 of 6.553 m. Most of the stable benchmarks are found in the central part of Singapore, i.e. situated at the Bukit Timah Granite formation – well known to be stable.



**Figure 3: Distribution of Stable PLBMs using STDBM6 of 6.553 m**

Figure 4 shows the distribution of the PLBMs using STDBM6 of 6.541 m. The 40 stable benchmarks are less distinct in terms of their distribution.

Thus, it is decided to re-adopt the pre-1997 value of the STDBM6 and to accept the 117 existing PLBMs as datum in the absolute-constraint least squares adjustment.



**Figure 4: Distribution of Good PLBMs using STDBM6 of 6.541 m**

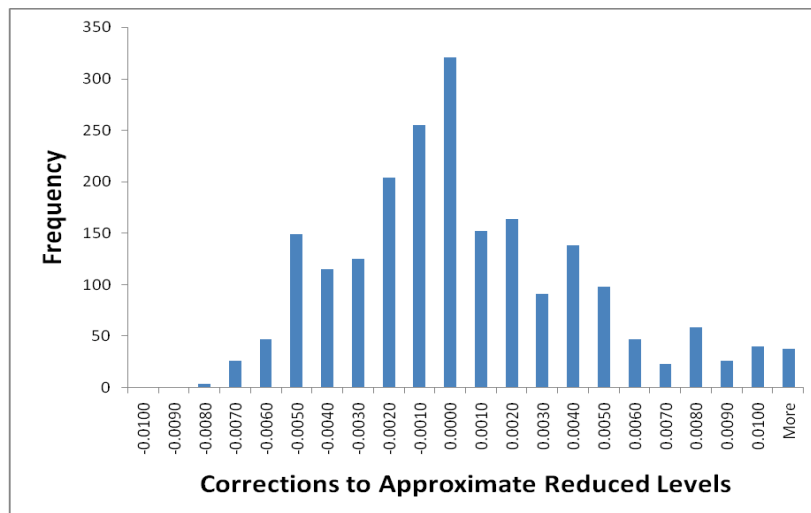
### 3. RESULTS OF LEAST SQUARES ADJUSTMETN

STDBM6 and the other 116 existing PLBMs were held fixed in a absolute (or fully) constraint least squares adjustment after a minimum constraint least squares adjustment which detected outlying observations. These bad observations were re-observed before the fully constraint solution. The parameters for the least squares adjustment carried out using Move3D are shown in Table 1.

**Table 1: Parameters for Least Squares Adjustment**

STATIONS	
Number of known stations	117
Number of unknown stations	3081
Total	3198
OBSERVATIONS	
Height differences	5608
Known coordinates	117
Total	5725
UNKNOWNNS	
Coordinates	3198
Total	3198
Degrees of freedom	2527

Figure 5 shows the distribution of the corrections to the approximate reduced levels of the benchmarks. The corrections are quite well distributed, approximating a normal curve distribution. No discernable biasness in the adjustment.

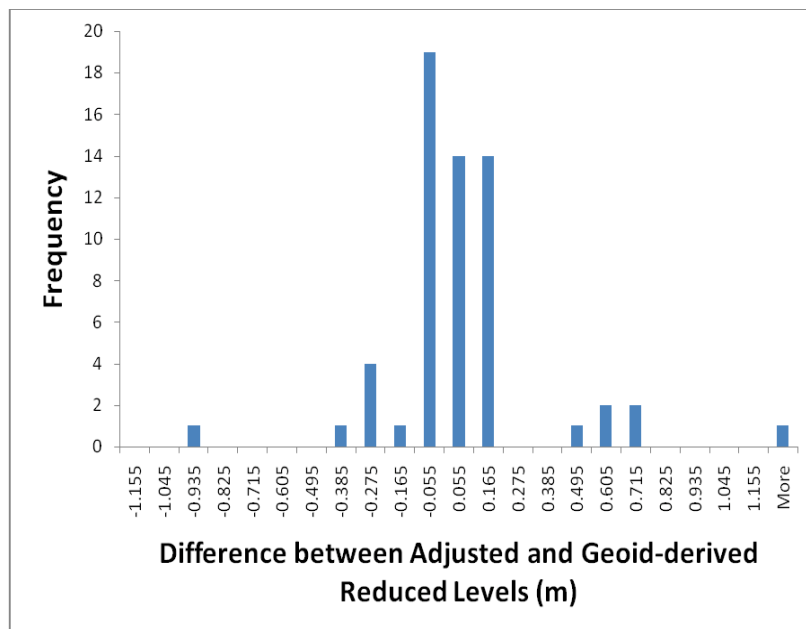


**Figure 5: Distribution of the Corrections to Approximate Reduced Levels**

#### 4. GEOMETRIC GEOID MODELLING

Stepwise Multiple Regression using the method of forward selection was used to compute the polynomial equation for the geometric geoid for the mainland of Singapore. The software used was StatGraphics Centurion XV Version 15.2 statistical software (Statpoint, 2009).

43 independent variables derived from the easting and northing coordinates of 464 PLBMs were made available to compute the multiple regression surface to best fit 464 PLBMs with geoid separations (GS) which are the dependent variables in the formulation. In the stepwise method, each independent variable is added one by one to the equation. Only the significant independent variables are retained in the progressive process. 58 benchmarks were found to be not able to fit in with the models. Figure 6 shows the distribution of the differences between the adjusted reduced levels (from precise leveling) and reduced levels derived from the geometric geoid of the 58 PLBMs which were rejected in the geoid modeling. 55% of the differences are about 0.05 m and about 22% have differences ranging from about 0.2 m to greater than 1 m.



**Figure 6: Distribution of the differences between adjusted and geoid-derived reduced levels of the 60 PLBMs rejected in the geoid modelling**

Only five (5) independent variables and a constant term were ultimately adopted as shown in Eq. (1):

$$GS = 8.94184 + 2.08529 * E - 0.16502 * N - 0.661429 * E^2 - 0.139884 * N^2 + 0.232462 * E^6 * N \dots\dots\dots(1)$$

E and N are normalized values of the easting and northing coordinates, i.e. ranging from 0 to 1.

## 5. VERIFICATION OF GEOMETRIC MODEL

The geometric geoid was verified using two test data set. The first set comprises 26 post-processed static GPS observations and 6 RTK observations. The differences between the adjusted reduced level as computed in this project and the reduced levels deduced from the geometric geoid ranges from -0.034 m to 0.059 m.

The 58 PLBMs which were previously rejected in the geoid modeling had their ellipsoidal heights re-observed using RTK techniques and used as the second test data set. The well distributed first (blue) and second (red) Test PLBMs are illustrated in Figures 7.

Errors were found in the previous RTK ellipsoidal heights of these 58 PLBMs. Figure 8 shows the distribution of the differences in the resurveyed RTK ellipsoidal heights. Differences of up to about - 0.4 m were found in the RTK heights. Other large errors (about 1 m and greater) as shown in Figure 6 were attributed to clerical errors.

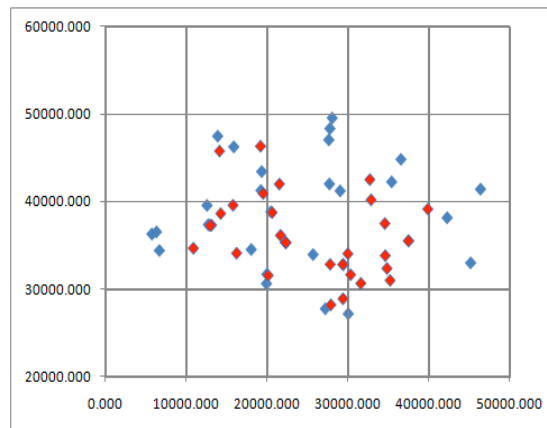


Figure 7: Distribution of First (Blue) and Second (Red) Test PLBMs

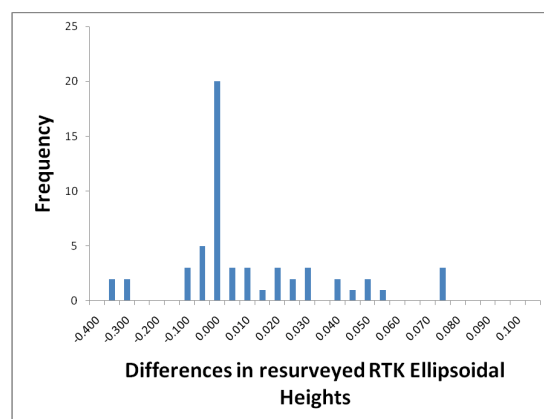
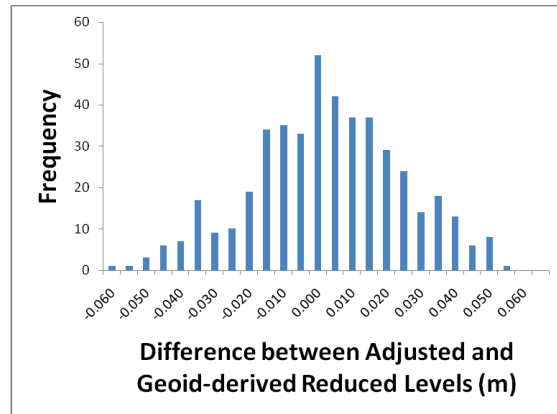


Figure 8: Distribution of the differences in resurveyed RTK ellipsoidal heights

Figure 9 illustrates the distribution of the differences between the adjusted and the geoid-derived reduced levels of all 464 PLBMs. About 82%, 95% and 99% of the PLBMs have their differences between the adjusted and geoid-derived reduced levels confined to within  $\pm 0.030$  m,  $\pm 0.040$  m and  $\pm 0.050$  m, respectively.



**Figure 8: Distribution of the differences between adjusted and geoid-derived reduced levels of 464 PLBMs**

## 6. RECOMMENDED PROCEDURE FOR RTK HEIGHTING

Two sets of testing were carried out to ascertain the optimum way of performing RTK heighting as it is a critical component in deriving reduced levels from the Geoid Model:

- (1) 5 RTK heights obtained with individual initialization; and
- (2) 5 RTK initializations with 3 RTK heights each.

In Set (1), the median of the 5 RTK heights is adopted as the ellipsoidal height. And for Set (2), the median of the medians of the 3 RTK heights is adopted. The purpose of re-initialization is to detect incorrect ambiguity resolution giving rise to wrong ellipsoidal height. A consideration on the choice of the optimum RTK observation, besides the results, would then rests on the time duration of the two tests. Both tests took between 20 minutes to slightly more than 30 minutes. So, the repeated observations in each initialization does not take more time than the initialization process which gives confidence to the correct ambiguity resolution in each RTK reading.

Both tests seem to give comparable results. The differences are within 30 mm. Thus, it is recommended to adopt 5 RTK initializations with 3 RTK heights each as the desirable method for the RTK heighting.



## 7. CONCLUSIONS

The geometric geoid, as determined using multiple regression forward stepwise method, RTK heighting and the precise levelling, is able to achieve accuracy to within  $\pm 0.030$  m,  $\pm 0.040$  m and  $\pm 0.050$  m for 82%, 95% and 99%, respectively.

It is recommended to perform 5 separate initializations and to acquire 3 readings in each, i.e. minimum of 15 reading in all for each benchmark. The middle value of the 5 middle values of the 5 sets of readings is to be used to derived reduced level from the ellipsoidal height and the geoid model.

The geometric geoid is suited engineering application which can accommodate  $\pm 5$  cm uncertainty in the height.

## REFERENCES

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## BIOGRAPHICAL NOTES

Assoc. Prof. Tor Yam Khoon joined the School of Civil & Environmental Engineering, Nanyang Technological University (NTU) in mid-'92. He was awarded the Bachelor of Land Surveying degree by the University of New South Wales, Australia in 1976 and a Master degree specializing in precise engineering survey in 1984. Dr. Tor obtained his Ph.D. from NTU in 2004. He had 16 years of industrial experience as a land surveyor with exposure in engineering survey, title survey, mapping survey and reclamation surveying before joining NTU.

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